SUBCRITICAL REGION OF LAMINAR ADIABATIC GAS FLOW IN A TWO-DIMENSIONAL CHANNEL

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It is shown that so-called developed laminar gas flow in a two-dimensional channel cannot continue beyond values of $\lambda \cong 0.74$. At $\lambda > 0.74$ the flow is accompanied by considerable deformation of the velocity profiles in the direction of greater fullness, while the resistance coefficient ζ increases.

In [1] Thestkov gives an integral method of computing the laminar flow of a gas in a two-dimensional channel based on the use of velocity profiles of the form*

$$u = u_1[2\eta/\eta_\delta - (\eta/\eta_\delta)^2], \quad (1)$$

where $\eta = \int_0^y \rho\, dy$ is the Dorodnitsyn variable, and $\eta_\delta = \eta|_{y=\delta}$ is the value of η at the boundary layer edge.

Let the velocity at the channel entrance be subsonic, so that the boundary layers merge at a sufficient distance from the entrance.

On the basis of the relations obtained in [2], we shall show that the assumptions made in [1] about the velocity profiles are physically warranted only up to certain values $\lambda < \lambda_r$. Subsequent flow in the region $\lambda_r < \lambda < \lambda_{cr}$ is accompanied by a substantial change in profile [1]. In the developed flow region $\delta = 1$, $\eta = \eta_1$, and the profile may be written in the form [1]

$$u = u_1[2H - H^2] = u_1[1 - (1 - H)^2], \quad (2)$$

where

$$H = \eta/\eta_1.$$

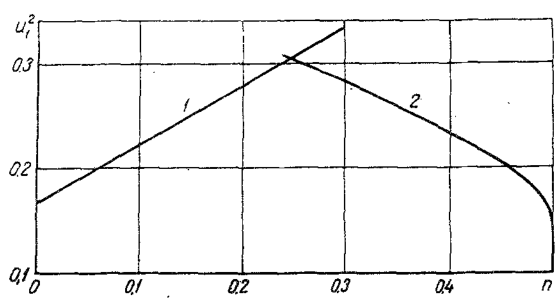

Fig. 1. Relation between the velocity on the axis u_1 and the parameter n: 1) at the instant of transition, and 2) in the precritical section.

Let us examine the more general family of profiles with parameter n ($0 < n < 1$)

$$u = u_1[1 - (1 - H)^{1/n}]. \quad (3)$$

*The notation is the same as in [2].

Profile (2) corresponds to $n = 1/2$.

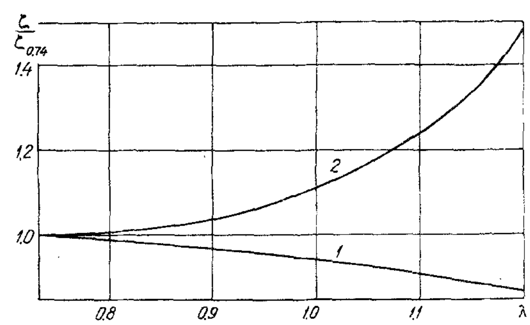

Fig. 2. Variation of resistance coefficient in the precritical section 1) without and 2) with account for deformation of the velocity profile.

As shown in [2], the velocity on the axis u_1 and the parameter n may be found from the system of quasi-linear equations

$$L_{11}\frac{du_1}{dx} + L_{12}\frac{dn}{dx} = L_1,$$
$$L_{21}\frac{du_1}{dx} + L_{22}\frac{dn}{dx} = L_2. \quad (4)$$

Here

$$L_{11} = I_{uu}\left(\frac{1}{I_{uu}}\frac{\partial I_{uu}}{\partial u_1} - \frac{1}{I_u}\frac{\partial I_u}{\partial u_1}\right) - u_1\frac{1 - I_{uu}}{1 - u_1^2},$$

$$L_{12} = I_{uu}\left(\frac{1}{I_{uu}}\frac{\partial I_{uu}}{\partial n} - \frac{1}{I_u}\frac{\partial I_u}{\partial n}\right),$$

$$L_{21} = \frac{2k}{k-1}\frac{u_1}{1 - u_1^2} - \frac{1}{I_u}\frac{\partial I_u}{\partial u_1} - \frac{1}{1 - I_{uu}}\frac{\partial I_{uu}}{\partial u_1},$$

$$L_{22} = \frac{1}{I_u}\frac{\partial I_u}{\partial n} - \frac{1}{1 - I_{uu}}\frac{\partial I_{uu}}{\partial n},$$

$$L_1 = \frac{I_u}{G_1}\tau_0(f - 1), \quad L_2 = -\frac{2k}{\kappa - 1}\frac{I_u}{G_1}(1 - I_{uu})\tau_0 f.$$

$$I_u = \int_0^1 u\, dH, \quad I_{uu} = \int_0^1 u^2 dH.$$

In these equations all the quantities are dimensionless. The parameter f is proportional to the curvature of the velocity profile on the channel axis

$$f = -\frac{\mu_1}{\tau_0}\left(\frac{\partial^2 u}{\partial y^2}\right)_1$$

and cannot be negative in view of its physical meaning. The quantities I_u and I_{uu} can easily be found using (3),

$$I_u = u_1/(n + 1), \quad I_{uu} = 2u_1^2/(n + 1)(n + 2). \quad (5)$$

The mean velocity u_m can be expressed in terms of the velocity on the axis u_1 and the parameter n,

$$u_m = I_{uu}/I_u = 2u_1/(n+2). \qquad (6)$$

Now let

$$n = \frac{1}{2}, \quad \frac{dn}{dx} = 0.$$

Taking into account that $du_1/dx > 0$, from the second equation of (4), we find that $f \geq 0$ when $L_{21} \leq 0$ and $f \leq 0$ when $L_{21} \geq 0$.

It is not difficult to see that (when k = 1.4)

$$L_{21} = 7u_1/(1-u_1^2) - 1/u_1 - 4u_1/(3.75 - 2u_1^2).$$

Thus it may easily be seen that $L_{21} \leq 0$ when $u_1 \leq u_{1r}$ and $L_{21} \geq 0$ when $u_1 \geq u_{1r}$, where u_{1r} is the root of the equation $L_{21} = 0$. Thus with n = 1/2 the flow cannot continue beyond the limit $u_1 > u_{1r}$. Putting $L_{21} = 0$, we find: $u_{1r}^4 - 2.333\, u_{1r}^2 + 0.312 = 0$, so $u_{1r}^2 \cong 0.1424$, $u_{1r} \cong 0.378$, which corresponds to $u_m \cong 0.303$ and the mean $\lambda_r \cong 0.74$. When $\lambda = \lambda_r = 0.74$, the quantity f becomes zero. Putting $f = 0$ for all $0.74 \leq \lambda \leq \lambda_{cr}$, as in [2], it is not difficult to find, from (4), the dependence of n and velocity u_1 on x. Confining ourselves to finding the dependence of n on u_1, we obtain the equation

$$dn/du_1 = -L_{21}/L_{22},$$

or in expanded form, after small transformations,

$$\frac{dn}{du_1^2} = \frac{n+2}{2u_1^2(1-u_1^2)} \times \qquad (7)$$

$$\times \frac{4u_1^2(1-u_1^2) - (8u_1^2 - 1)[(n+1)(n+2) - 2u_1^2]}{(n+2)^2 + 2u_1^2}.$$

This equation must be integrated, beginning from $u_1^2 = 0.1424$ when n = 0.5. The integration is carried out up to transition, i.e., until $du_1/dx \to \infty$. At the critical section we must have

$$\begin{vmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{vmatrix} = 0.$$

After transformation we have

$$u_1^4 - au_1^2 + b = 0, \qquad (8)$$

where

$$a = \frac{7}{12}(n+1)(n+2),$$

$$b = \frac{(n+1)(n+2)}{12} + \frac{(n+2)^2}{24}[(n+1)(n+2) - 2].$$

Curve 2 in Fig. 1 was obtained by numerical integration of (7). As u_1 grows, the parameter n decreases sufficiently rapidly to indicate a substantial deformation of the velocity profile, the profile becoming fuller as u_1 increases.

Let us examine the change in coefficient of resistance

$$\zeta = \frac{8\mu_0}{\widetilde{\mathrm{Re}} G_1 u_m}\left(\frac{\partial u}{\partial y}\right)_0$$

in the precritical section. We find the ratio of ζ for $u_1 > u_{1r} = 0.328$ to the quantity $\zeta_{0.74}$ at the beginning of the precritical section:

$$\frac{\zeta}{\zeta_{0.74}} = \frac{u_m^{(0.74)}}{u_m}\left(\frac{\partial u}{\partial y}\right)_0 \Big/ \left(\frac{\partial u}{\partial y}\right)_0^{(0.74)},$$

where the index 0.74 indicates that the corresponding quantity must be determined for $\lambda = 0.74$. Using equality (6) and noting, in addition, that

$$\left(\frac{\partial u}{\partial y}\right)_0 = \rho_0\left(\frac{\partial u}{\partial \eta}\right)_0 = p\left(\frac{\partial u}{\partial \eta}\right)_0 = \frac{p}{\eta_1}\frac{u_1}{n}$$

and

$$p/\eta_1 = 1 - I_{uu} = 1 - 2u_1^2/(n+1)(n+2),$$

we find, at the beginning of the precritical section, when n = 0.5, $\lambda = 0.74$, $u_1 = 0.328$, that

$$u_m^{(0.74)} = \frac{2}{2.5}\cdot 0.328 = 0.262,$$

$$\left(\frac{\partial u}{\partial y}\right)_0 = \left(1 - \frac{2\cdot 0.1424}{1.5\cdot 2.5}\right)2\cdot 0.328 = 0.605.$$

Therefore,

$$\frac{\zeta}{\zeta_{0.74}} = \frac{n+2}{4.62}\left[1 - \frac{2u_1^2}{(n+1)(n+2)}\right]. \qquad (9)$$

If the velocity profile remained unchanged in the precritical section, i.e., if n were 1/2, it would follow from (9), as in [1], that the coefficient ζ decreases (curve 1 in Fig. 2). By substituting into (9) values of n found after integration of (7), we can verify that the resistance coefficient increases (curve 2 of Fig. 2).

Thus, the conclusions reached in [2] for turbulent flow remain qualitatively the same for laminar flow.

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